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REALIZATION OF THE CONSTANT-RESISTANCE RC LATTICE WITH ACTIVE ELEMENTS

A THESIS

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REALIZATION OF THE CONSTANT-RESISTANCE

RC LATTICE WITH ACTIVE ELEMENTS

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Date Approved by Chairman May 10, 1962

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SUMMARY

Cascade synthesis of active RC networks offers many attractive advantages--such as ease in alignment and simplicity of the networks. One important method of cascade synthesis is the use of constant-resistance lattices. The passive constant-resistance lattice synthesis technique is extended in this investigation to active RC networks. The circuit elements used in the active lattices are positive capacitances, positive and negative resistances, and gyrators.

Two general synthesis procedures are presented for synthesizing the transfer function of biquadratic form. Synthesis Method I is capable of realizing a pair of unrestricted zeros and a pair of left-halfplane poles. Synthesis Method II is capable of realizing a pair of unrestricted zeros and poles. Each synthesis method generally requires four active elements: two gyrators and two negative resistors. The transfer functions for the constant-resistance RC lattices are realized so that they may be unbalanced.

In special cases when the transfer function contains only real poles and zeros the unbalanced RC lattice requires only one gyrator. In these cases the lattice elements are specified in terms of the coefficients of the transfer function. In other cases when the transfer function is of order less than biquadratic the network is also specified.

Several alternative procedures, in which the unbalanced networks are specified in advance, are also presented. These procedures are applicable only to transfer functions of restricted character.

CHAPTER I

INTRODUCTION

Many techniques have been devised for the synthesis of networks utilizing resistances (R), capacitances (C), and various active elements. The purpose of this investigation is to develop general synthesis procedures to realize transfer functions with active elements for the constant-resistance RC lattice. The active elements will be limited to negative resistances and gyrators.

Thomas has published what is apparently the only article on the active RC lattice[1]. His method is completely general and is capable of realizing a pair of unrestricted zeros and a pair of left halfplane poles. For each biquadratic factor (ratio of two quadratics in s) of a transfer function, two negative impedance converters are required. By means of this method, it is possible to unbalance the symmetrical constant-resistance lattice to an equivalent twoport.

The gyrator is a passive nonreciprocal twoport device which has been extensively developed in the past few years[2]. For the purposes of this analysis the gyrator is considered as an active element which can be treated as a separate entity. It is characterized by the following chain matrix

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0 & R \\ G & 0 \end{bmatrix}, \text{ where } GR = 1 .$$

Thus, the input immittance at one port of a gyrator is the reciprocal of the terminating immittance at the other port.

Similarly, a negative impedance converter is an active twoport device which is characterized by the following chain matrix.

$$\begin{bmatrix} \pm 1 & 0 \\ 0 & \pm 1 \end{bmatrix} .$$

Therefore, the input immittance is the negative of the load immittance.

It is of interest to determine if there are alternative active constant-resistance lattices utilizing active elements other than the procedure presented by Thomas. As a first approach to this problem the active elements were restricted to negative resistances only. It was found, however, that it is not possible to realize any transfer impedance for the constant-resistance lattice utilizing $\pm R$ and $+C$ only. As a second approach, the active elements were restricted to one negative impedance converter. It was found that only real poles and real zeros could be realized by the transfer impedance. In the third approach, the active elements were extended to include gyrators and negative resistances. It was found that it is possible to realize with these active elements transfer functions of the biquadratic form as a constant-resistance lattice. The lattice is realized so that it could be unbalanced. In special cases where the transfer function was less than biquadratic the solutions are tabulated.

CHAPTER II

SYNTHESIS METHODS FOR GENERAL TRANSFER FUNCTIONS

In order to realize any transfer function [transfer impedance Z_{12}, transfer admittance Y_{12}, voltage ratio E_2/E_1, or current ratio I_2/I_1] by a cascade of constant-resistance sections, it is sufficient to show a method for realizing a general biquadratic factor for the transfer impedance Z_{12}. Such a factor can be written as

$$Z_{12} = (k)\frac{bs^2 + b_1 s + b_o}{s^2 + a_1 s + a_o} \qquad (1)$$

To realize this transfer impedance in the form of a constant-resistance lattice requires that

$$Y_a = \frac{1}{Z_a} = Z_b = \frac{1 + Z_{12}}{1 - Z_{12}} \,. \qquad (2)$$

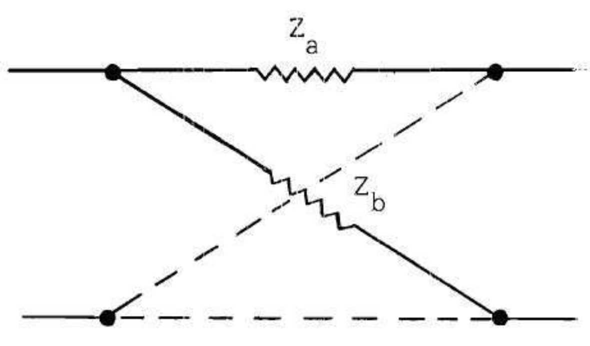


Figure 1. A Symmetric Lattice

Substituting Equation (1) into Equation (2) gives

$$Y_a = Z_b = \frac{(1 + kb)s^2 + (a_1 + kb_1)s + (a_o + kb_o)}{(1 - kb)s^2 + (a_1 - kb_1)s + (a_o - kb_o)} . \tag{3}$$

<u>Synthesis Method I</u>.--One general method of realizing the transfer impedance Z_{12} by a constant-resistance RC section is to require that Z_b have only real negative poles. To ensure that Z_b meets this requirement, the following three conditions must be fulfilled.

a. $$\frac{a_1 - kb_1}{1 - kb} \tag{4}$$

b. $$(a_1 - kb_1)^2 > 4(1 - bk)(a_o - kb_o) > 0 \tag{5}$$

The restrictions under conditions (a) and (b) are necessary to ensure that Z_b be expressible as an RC impedance.

c. $$a_1 > 0, \quad a_o > 0 \tag{6}$$

Restriction (c) is necessary to ensure that the poles of Z_{12} are located in the left halfplane.

If all the coefficients are positive, a gain factor k can always be chosen to satisfy the requirements in Equations (4) and (5). It is interesting to note that k is not necessarily restricted to positive values. A negative k would only indicate that the output was 180° out of phase with the input. Furthermore, it is possible under special conditions that a value of k would not exist that would satisfy the requirements under (4) and (5). The transfer function in this case would not be p-r and, in general, is not of interest.

After a gain factor k has been chosen to fulfill the requirements in Equations (4) and (5), the function in (3) may be written as

$$Y_a = Z_b = \frac{As}{s + \sigma_1} + \frac{B}{s + \sigma_2} \pm R .$$

In this function and in similar functions to follow, the symbols A, B, R, σ_1, and σ_2 will represent real positive numbers. Now, Z_b can be realized with $\pm R$, C, and a gyrator. Similarly, in the Y_a branch a gyrator will be required. The constant-resistance RC lattice will have the structure shown in Figure 2. It can be shown, by standard methods of lattice reduction (Figure 3) that the lattice of Figure 2 can be unbalanced into the equivalent twoport of Figure 4.

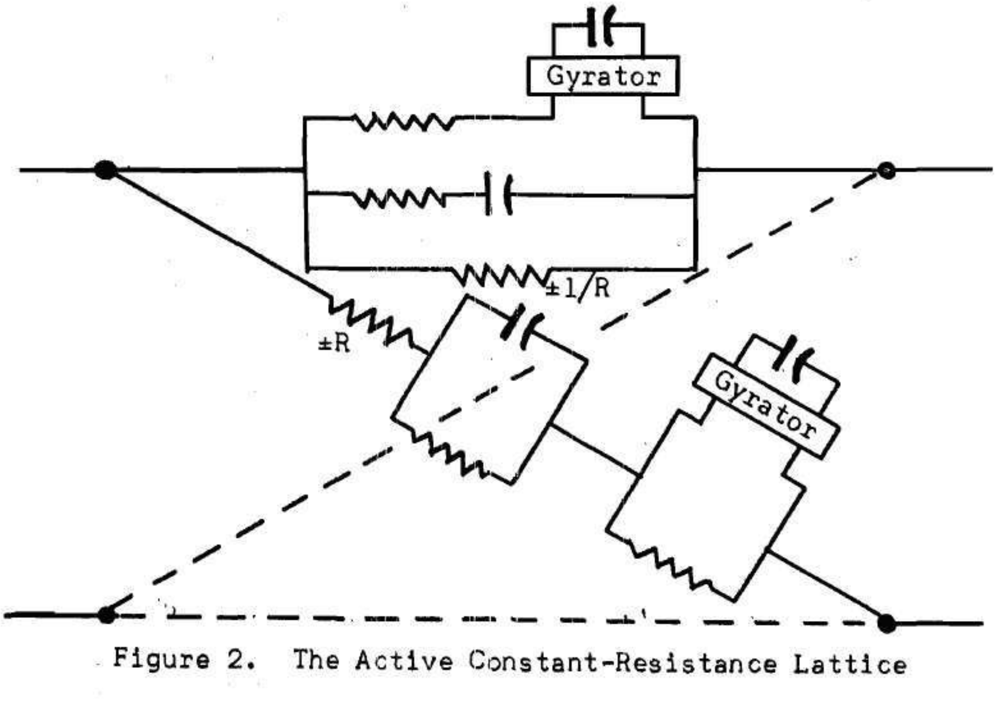


Figure 2. The Active Constant-Resistance Lattice

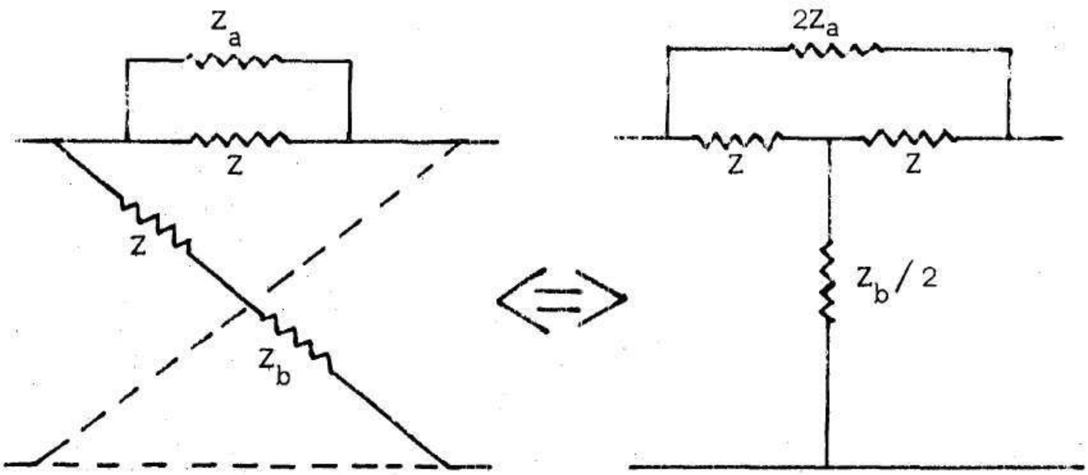


Figure 3. Reducing the Symmetrical Lattice to Its Unbalanced Equivalent.

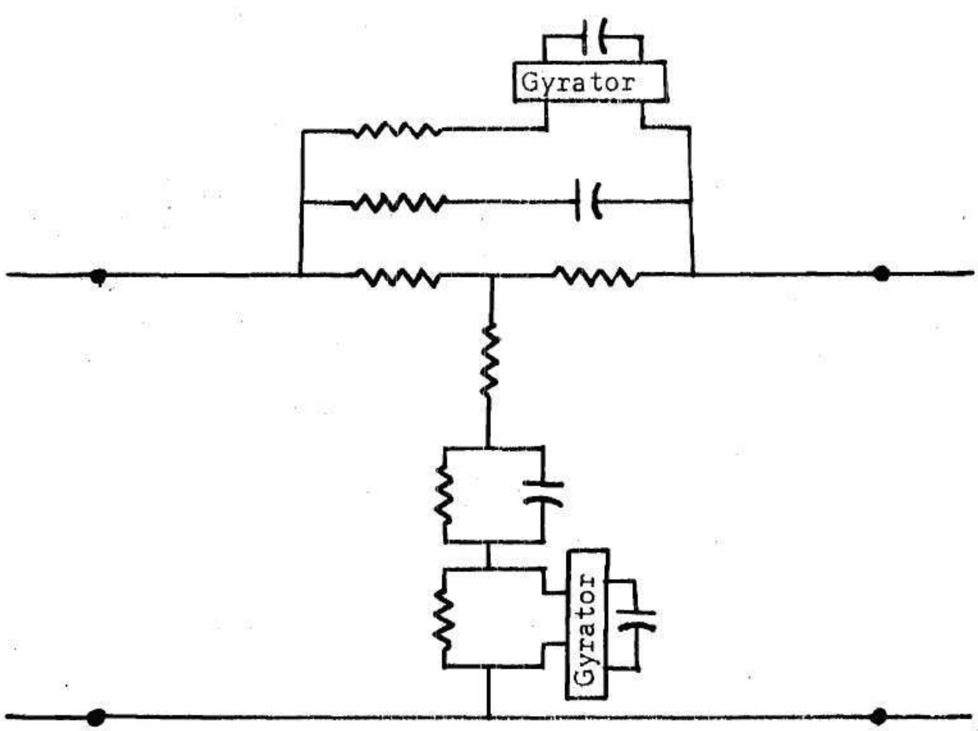


Figure 4. The Unbalanced Equivalent of Lattice of Figure 2

As an example, let

$$Z_{12} = k \frac{s^2 + 4s + 8}{s^2 + 8s + 32} . \tag{8}$$

This transfer impedance has complex poles and zeros in the left half-plane as shown in Figure 5. Equation (3) gives

$$Y_a = Z_b = \frac{(1 + k)s^2 + (8 + 4k)s + (32 + 8k)}{(1 - k)s^2 + (8 - 4k)s + (32 - 8k)} \tag{9}$$

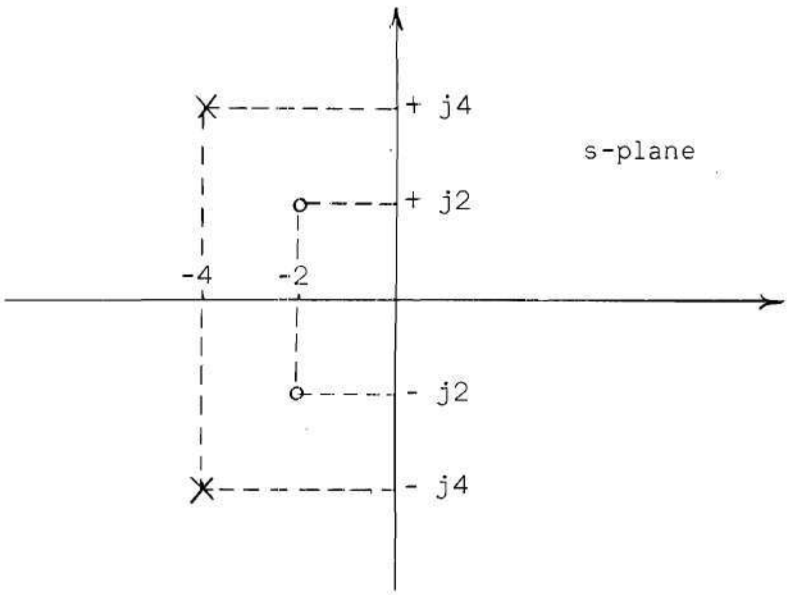


Figure 5. Location of Left-Halfplane Poles and Zeros for Transfer Impedance (8).

Selecting a gain factor k equal to 7/8 to satisfy conditions (4) and (5), we have

$$Y_a = Z_b = \frac{s15.97}{s + 29.14} + \frac{17.35}{s + 6.86} - 0.968 . \tag{10}$$

A possible realization of this transfer impedance is shown in Figure 6.

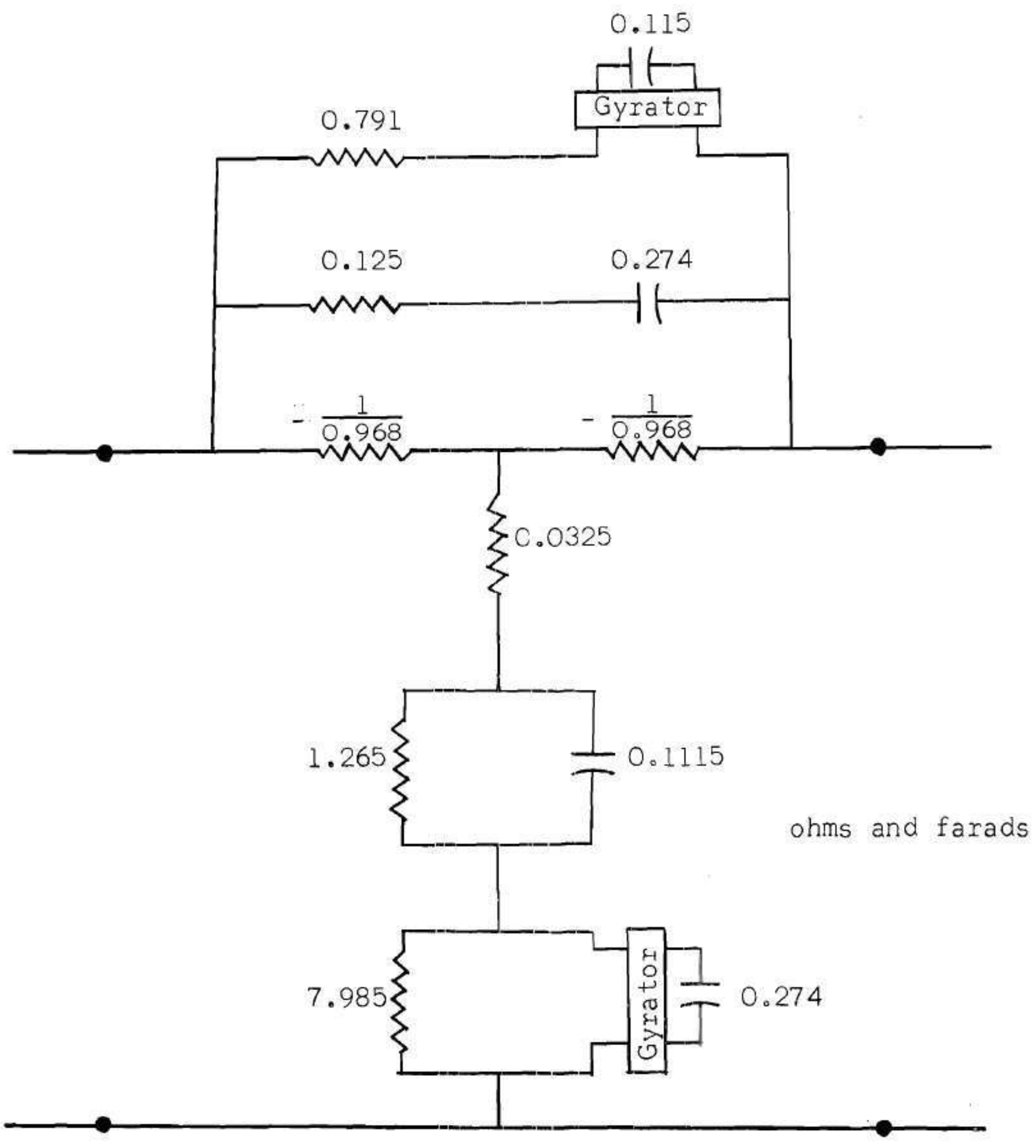


Figure 6. A Realization of the Transfer Impedance (8) with k Equal to 7/8.

Synthesis Method II.--An alternative general procedure to realize the transfer impedance in Equation (1) is to require that the function of (3) have one negative pole and one positive pole. (The condition of a positive pole for Z_b does not necessarily affect the stability of the transfer function.) To ensure that Z_b meets this requirement, one of the following conditions must be fulfilled.

$$(1 - kb) < 0 , \tag{11}$$

or

$$(a_o - kb_o) < 0 . \tag{12}$$

The selection of a gain factor k is simplified in this method as it has to satisfy only the restrictions in either Equation (11) or Equation (12) (but not both). In the case that restriction (11) is used, the function of Equation (3) may be written as

$$Y_a = Z_b = \frac{A}{s + \sigma_1} - \frac{Bs}{s - \sigma_2} \pm R . \tag{13a}$$

In the case that restriction (12) is used then the function in Equation (3) may be written as

$$Y_a = Z_b = \frac{As}{s + \sigma_1} + \frac{B}{s - \sigma_2} \pm R . \tag{13b}$$

The functions in Equation (13a) can be realized by the branches of Figure (7) and the desired Z_{12} is obtained. Similarly the functions in Equation (13b) are realized in Figure (8).

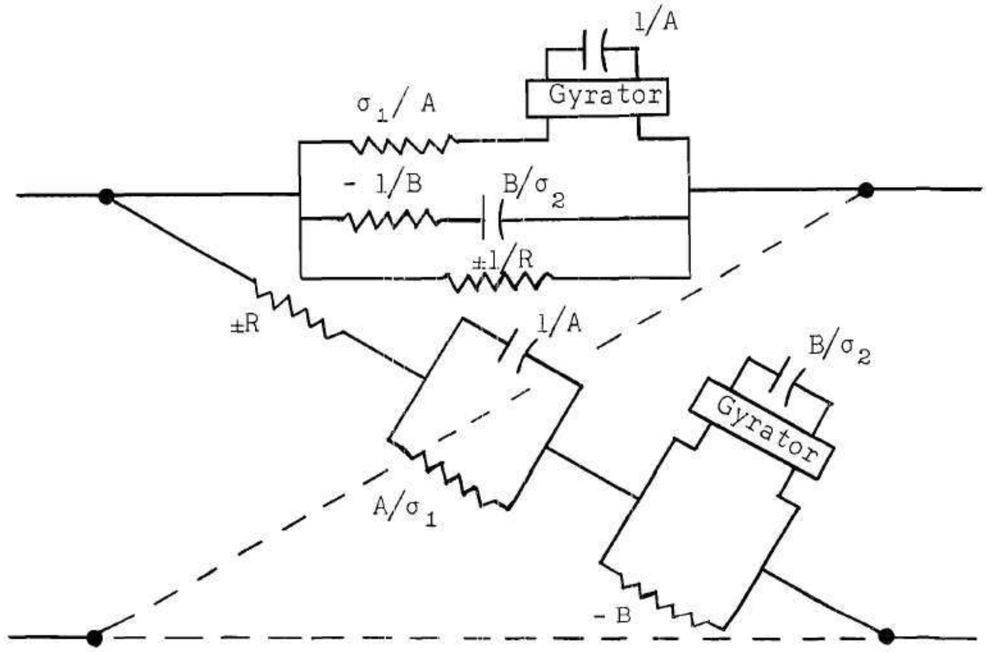


Figure 7. The Active Constant-Resistance ± RC Lattice for Function (13a)

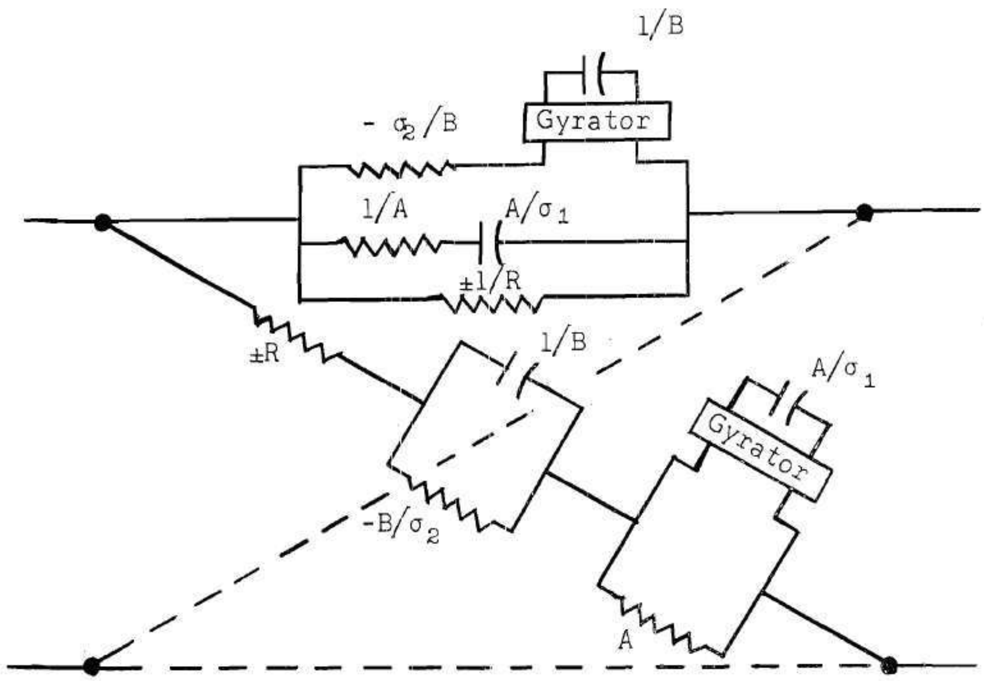


Figure 8. The Active Constant-Resistance ± RC Lattice for Function (13b)

As an example of Synthesis Method II, the transfer impedance of Equation (8) will again be realized. A gain factor between the limits $4 > k > 1$ will satisfy the condition given in Equation (11). If $k = 3$, then

$$Y_a = Z_b = -\frac{2s^2 + 10s + 28}{s^2 + 2s - 4} \; . \tag{14}$$

Further,

$$Y_a = Z_b = \frac{3.71}{s + 3.24} - \frac{7.8s}{s - 1.24} + 5.86 \; . \tag{15}$$

The unbalanced equivalent twoport is shown in Figure 9.

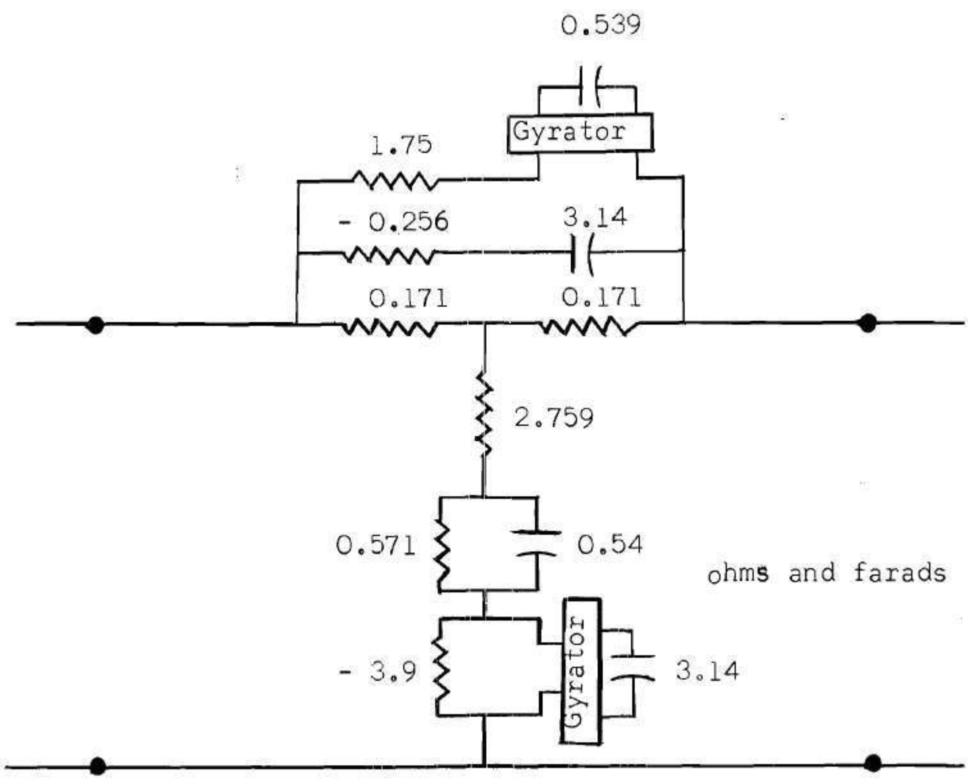


Figure 9. Realization of the Transfer Impedance for Example of Synthesis Method II

As a second example of Synthesis Method II let

$$Z_{12} = k \frac{s^2 + 8s + 32}{s^2 + 4s + 8}$$

which is the inverse of the function in Equation (8). A gain factor between the limits $1 > k > 1/4$ will satisfy the condition in Equation (12) and violate the condition in Equation (11). Therefore, let $k = 3/4$ hence

$$Y_a = Z_b = \frac{7s^2 + 40s + 128}{s^2 - 8s - 64},$$

or

$$Y_a = Z_b = \frac{101.67}{s - 12.95} + \frac{1.15s}{s + 4.95} + 5.84$$

The unbalanced equivalent twoport is shown in Figure (10).

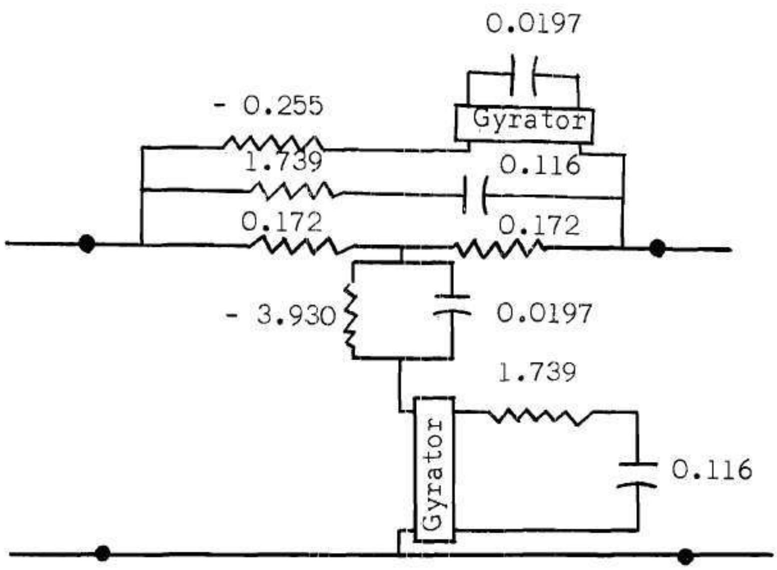
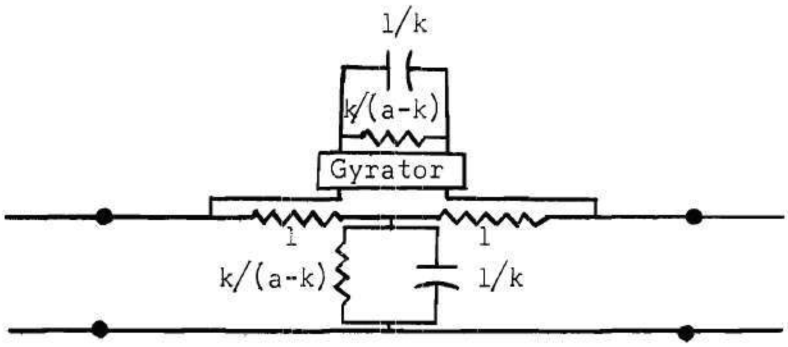


Figure 10. Network Realizing Transfer Impedance for the Second Example of Synthesis Method II.

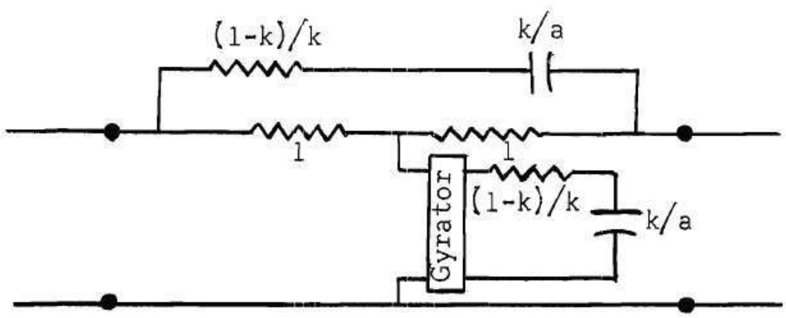
Special Cases.--When realizing transfer functions of order less than biquadratic form the circuit can be simplified and tabulated. The special cases illustrated are not necessarily the best solutions for realizing a given transfer function. This is because the symmetrical lattice was unbalanced by removing a one-ohm resistor from the branch impedances Z_a and Z_b. It is entirely possible that a better solution can be obtained by using the methods indicated in Synthesis Method I or Synthesis Method II. The following are special transfer functions.

Case A. $$Z_{12} = \frac{k}{s + a}$$



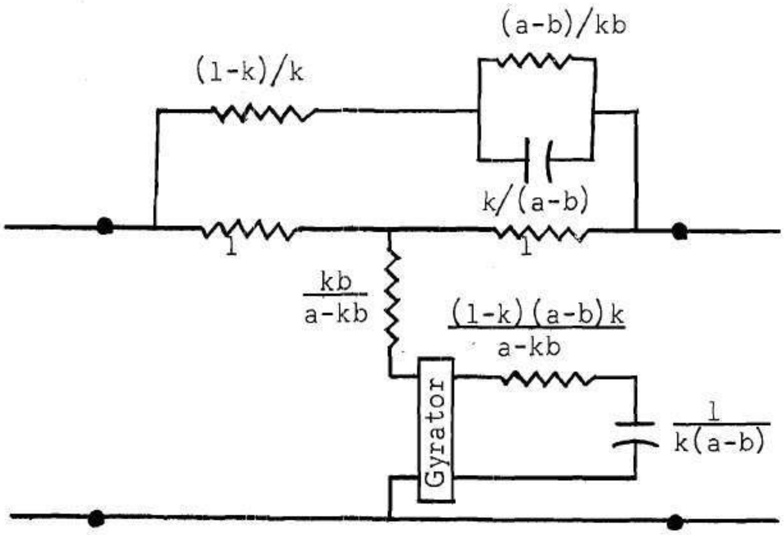
Restriction $k \leq a$

Case B. $$Z_{12} = \frac{ks}{s + a}$$



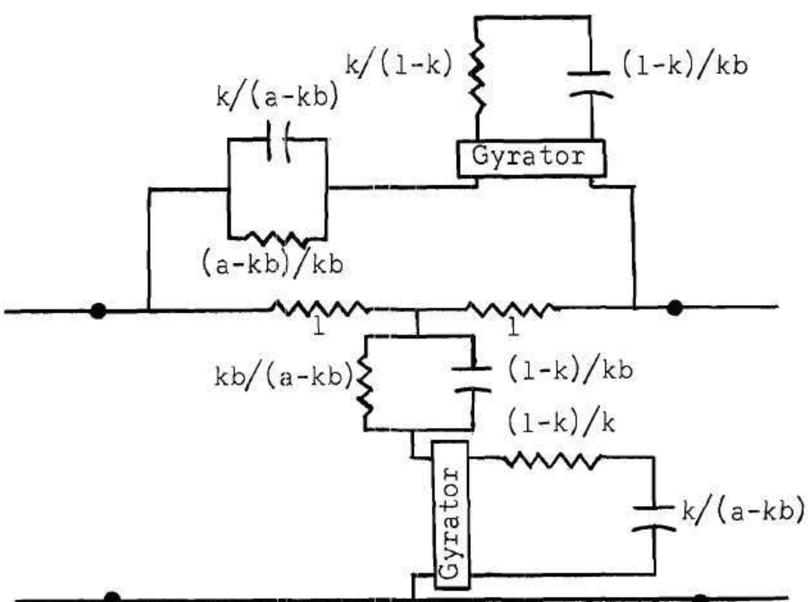
Restriction $k \leq 1$

<u>Case C</u>. $$Z_{12} = (k)\,\frac{s+b}{s+a}$$



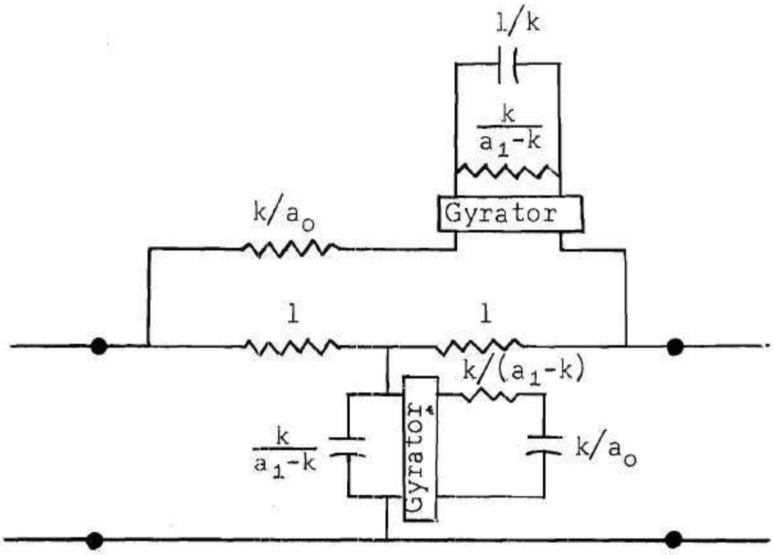
Restrictions $k \leq 1$, $a > b$

<u>Case D</u>. $$Z_{12} = (k)\,\frac{s+b}{s+a}$$



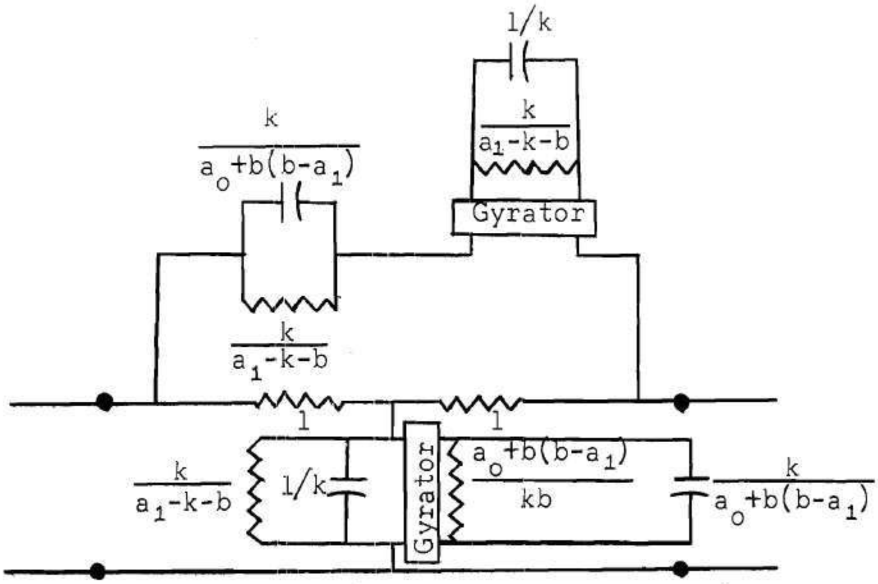
Restrictions $k \leq 1$, $k < a/b$

<u>Case E</u>.
$$Z_{12} = \frac{ks}{s^2 + a_1 s + a_0}$$



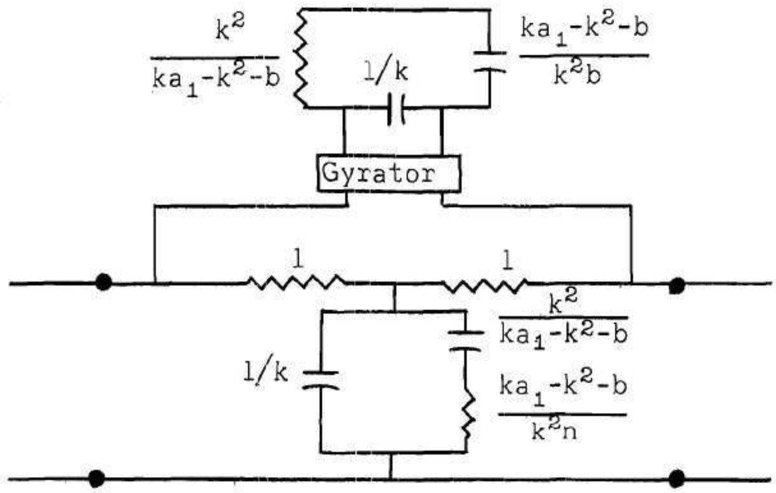
Restrictions $0 < k \leq a_1$

<u>Case F</u>.
$$Z_{12} = \frac{k(s + b)}{s^2 + a_1 s + a_0}$$



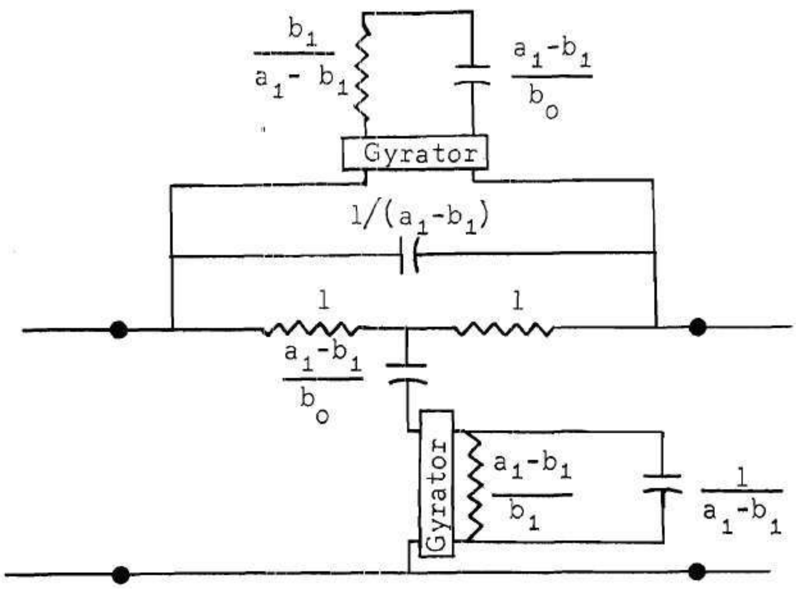
Restrictions $k \leq a_1$, $k \leq a_0/b$, $k \leq a_1 - b$ and $b^2 + a_0 < a_1 b$

Case G. $$Z_{12} = \frac{k(s+b)}{s^2 + a_1 s + a_o}$$



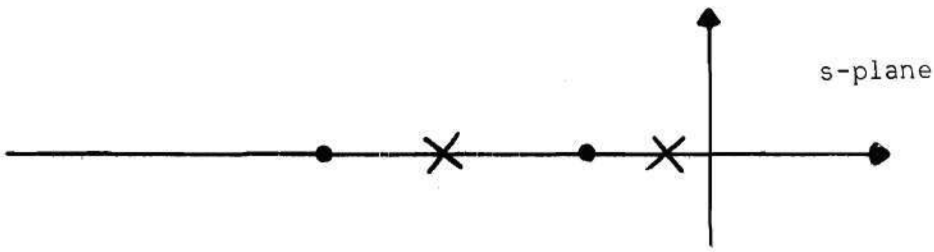
Restrictions $k = a_o/b$, and $ka_1 > k^2 + b$

Case H. $$Z_{12} = (k)\frac{s^2 + b_1 s + b_o}{s^2 + a_1 s + a_o}$$



Restrictions $k = 1$, $b_o = a_o$, $a_1 > b_1$

Case I. The realization of Z_{12} can be simplified when it contains only real poles and zeros that alternate along the real axis in the sequence as indicated below.

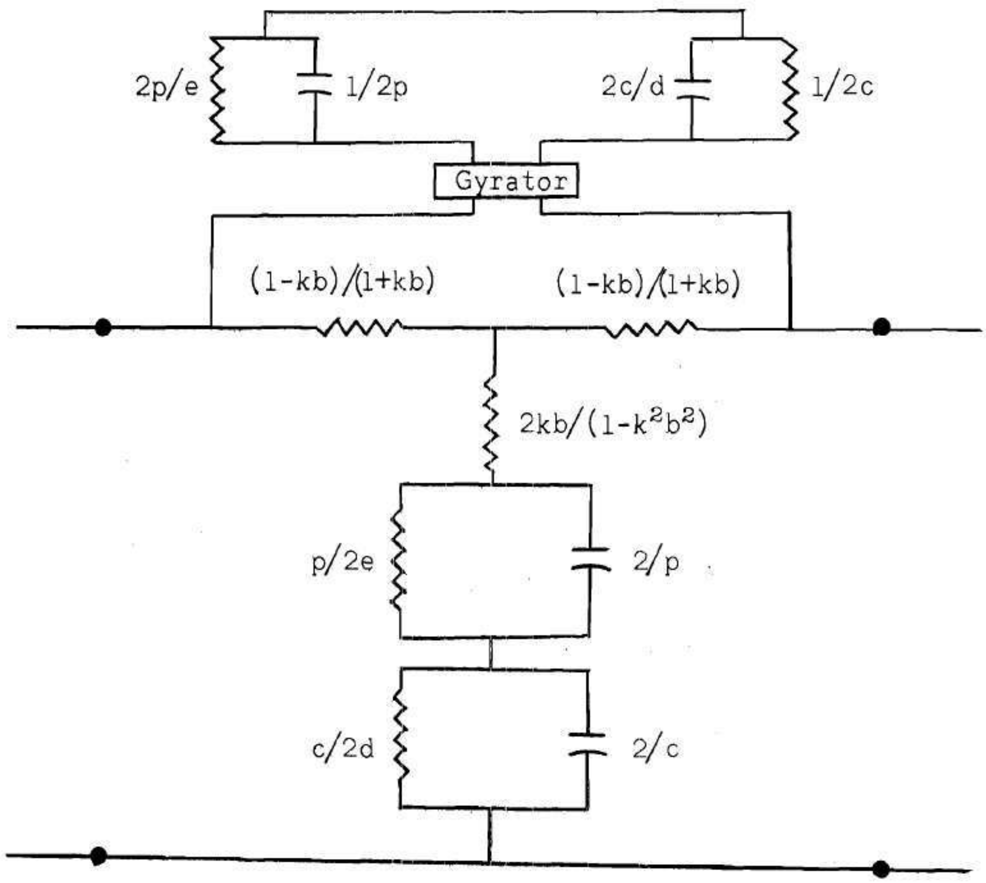


The desired unbalanced constant-resistance lattice can be specified with only one gyrator located in the Z_a branch. This method can realize real right-halfplane as well as left-halfplane poles and zeros for the transfer function since there is no restriction on the coefficients of Z_{12}. The only necessary condition that must be fulfilled is that the gain factor k must be chosen so that the following expression be satisfied.

$$(a_1 - kb_1)^2 - 4(a_o - kb_o)(1 - kb) > 0 .$$

This condition is necessary to ensure that the roots of Z_a and Z_b are real.

After a gain factor k is selected to satisfy the above condition, the network can be specified as follows.



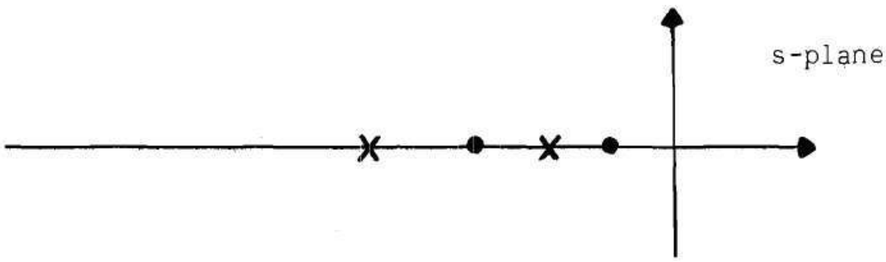
Where $M = \dfrac{1 + kb}{1 - kb}$, $N = \dfrac{a_1 + kb_1}{1 - kb}$, $P = \dfrac{a_o + kb_o}{1 - kb}$,

$Q = \dfrac{a_1 - kb_1}{1 - kb}$, $R = \dfrac{a_o - kb_o}{1 - kb}$, $p = N - MQ - c$,

$c = \dfrac{P - MR - (N - MQ)d}{\sqrt{Q^2 - 4R}}$, $d = \dfrac{Q}{2} - \dfrac{1}{2}\sqrt{Q^2 - 4R}$,

$e = \dfrac{Q}{2} + \dfrac{1}{2}\sqrt{Q^2 - 4R}$.

Case J. Similarly, the realization of Z_{12} can be simplified when it contains real zeros and poles that alternate in the following manner.

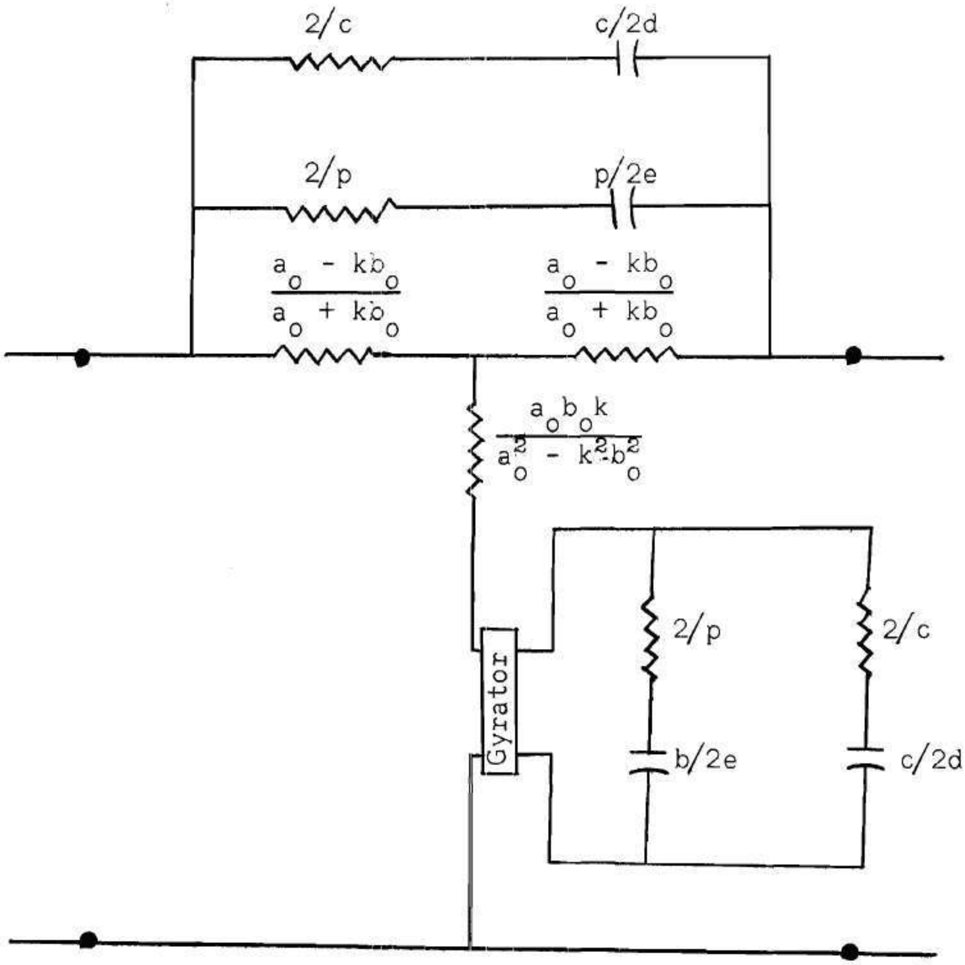


The desired unbalanced lattice can be specified with only one gyrator located in the Z_b branch. As in Case I, real right-halfplane and left-halfplane poles and zeros can be realized provided the gain factor k satisfies the inequality

$$(a_1 - kb_1)^2 - 4(a_o - kb_o)(1 - kb) > 0 .$$

This condition is necessary to ensure that the roots of Z_a and Z_b are real.

Thus, after a gain factor k is selected to satisfy the above condition, the unbalanced network can be specified as follows.



Where $M = \dfrac{1 + kb}{1 - kb}$, $N = \dfrac{a_1 + kb_1}{1 - kb}$, $P = \dfrac{a_o + kb}{1 - kb}$,

$Q = \dfrac{a_o - kb_o}{1 - kb}$, $R = \dfrac{a_1 - kb_1}{1 - kb}$, $A = \dfrac{P}{Q}$,

$p = (M - A) - c$, $e = \dfrac{R}{2} + \dfrac{1}{2}\sqrt{R^2 - 4Q}$,

$c = \dfrac{N - AR - (M - A)d}{\sqrt{R^2 - 4Q}}$, $d = \dfrac{R}{2} - \dfrac{1}{2}\sqrt{R^2 - 4Q}$.

CHAPTER III

SYNTHESIS METHODS FOR SPECIAL TRANSFER FUNCTIONS

A symmetrical lattice can be readily unbalanced if the branch impedances Z_a and Z_b can be realized as in Figure 11.

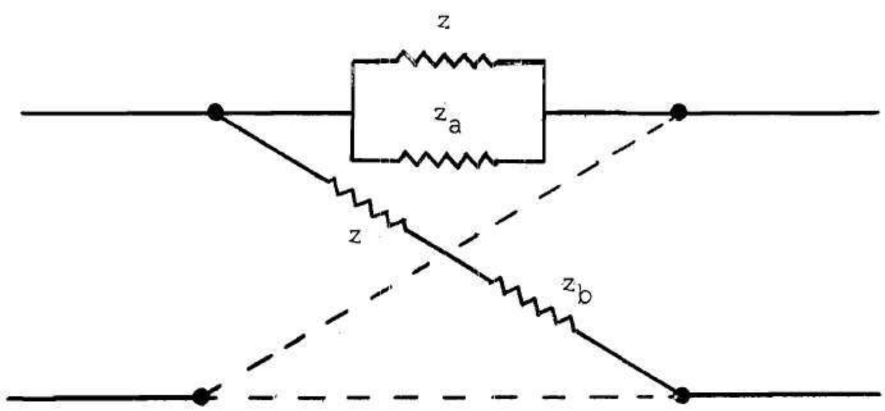


Figure 11. An Unbalanceable Symmetrical Lattice

For realization of this symmetrical lattice as a constant-resistance lattice requires that

$$\frac{1}{Z_a} = Y_a = Z_b = \frac{1 - Z_{12}}{1 + Z_{12}} \ . \qquad (18)$$

This will require

$$Z_a = \frac{z\ z_a}{z + z_a} \qquad (19)$$

and

$$Z_b = z + z_b \ . \qquad (20)$$

Now the function in (18) may be written as

$$Y_a = Z_b = \frac{z + z_a}{z\, z_a} = z + z_b \;. \tag{21}$$

By making $z_b = z_a$, Equation (21) yields

$$z = 1/z_a = 1/z_b \;. \tag{22}$$

The conditions in Equation (22) must be fulfilled in order that the symmetrical lattice of Figure 8 represent a constant-resistance lattice.

For the constant-resistance lattice to be realized by RC impedances requires that z_a be an RC impedance. This makes z an RL impedance. However, the RL impedance requirement may be removed by unbalancing the lattice with the use of two gyrators as shown in Figure 12.

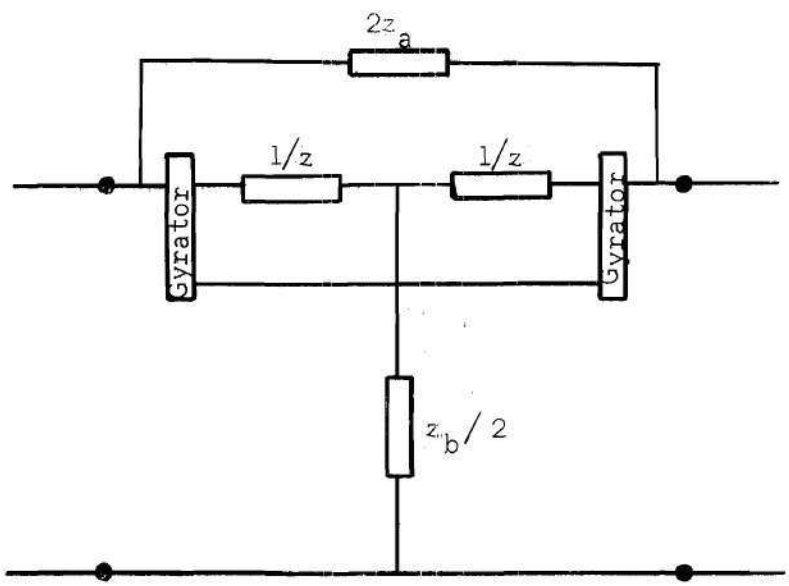


Figure 12. The Unbalanced Constant-Resistance RC Lattice

The branch impedance Z_b can now be expressed in terms of z_b.

$$Z_b = z_b + 1/z_b = \frac{z_b^2 + 1}{z_b} . \tag{23}$$

From Equation (18),

$$Z_{12} = \frac{z_b^2 - z_b + 1}{z_b^2 + z_b + 1} . \tag{24}$$

It is now desirable to determine the pole-zero restrictions on the Z_{12} function with regard to a realizable z_b.

<u>Synthesis Method III</u>.--A realizable RC impedance is

$$z_b = \frac{A}{s + B} + C \tag{25}$$

or

$$z_b = \frac{Cs + E}{s + B} \tag{26}$$

where

$$E = (A + CB) . \tag{27}$$

From Equation (24)

$$Z_{12} = \frac{(C^2 - C + 1)s^2 + (2CE + 2B - BC - E)s + (B^2 + E^2 + BE)}{(C^2 + C + 1)s^2 + (BC + E + 2CE + 2B)s + (B^2 + E^2 + BE)} \tag{28}$$

The pole-zero locations for the Z_{12} function can be specified as

$$s_p = - \sigma_p \pm j \, \omega_p \quad \text{poles} \tag{29}$$

and

$$s_z = - \sigma_z \pm j \, \omega_z \quad \text{zeros} \tag{30}$$

Therefore:

$$\omega_p = \frac{0.866\ A}{C^2 + C + 1}\ , \tag{31}$$

$$\sigma_p = \left\{\frac{B(C^2 + C + 1) + AC + A/2}{C^2 + C + 1}\right\}\ , \tag{32}$$

$$\omega_z = \frac{0.866\ A}{C^2 - C + 1}\ , \tag{33}$$

and

$$\sigma_z = \left\{\frac{B(C^2 - C + 1) + AC - A/2}{C^2 - C + 1}\right\}\ . \tag{34}$$

Since there are four specified quantities ω_p, σ_p, ω_z, and σ_z and only three variables A, B, and C, a general Z_{12} function can not be realized exactly, except in special cases.

The three variables A, B, and C can be specified in terms of ω_p, ω_z, σ_p, and σ_z.

$$C = \frac{\omega_z + \omega_p - \omega_p\ \omega_z}{\omega_p - \omega_z} \tag{35}$$

$$A = \frac{\omega_p\ \omega_z}{0.433}\left\{\frac{\omega_z + \omega_p - \omega_p\ \omega_z}{(\omega_p - \omega_z)^2}\right\} > 0 \tag{36}$$

or

$$A = \frac{\omega_p\ \omega_z}{0.433}\ \frac{C}{(\omega_p - \omega_z)} > 0\ . \tag{37}$$

The real portion of the pole σ_p or zero σ_z can be used to determine the variable B (but not both).

$$B = \frac{\sigma_p(C^2 + C + 1) - A(C + 1/2)}{C^2 + C + 1} \tag{38}$$

or

$$B = \frac{\sigma_z(C^2 - C + 1) + A(C - 1/2)}{C^2 - C + 1} . \tag{39}$$

This method of realizing Z_{12} can realize exactly three of the four specified quantities. ω_p and ω_z are always determined exactly by A and C. (Reference Equations (35) and (36).) However, A must not be negative. A negative A would necessitate the use of negative capacitance, which is not realizable by this method. This places further restrictions on ω_p and ω_z. In order to ensure that A will be positive requires either

$$\omega_p < \frac{\omega_z}{\omega_z - 1} \tag{40a}$$

or

$$\omega_z < \frac{\omega_p}{\omega_p - 1} ; \tag{40b}$$

but not both.

The location of the real of the poles σ_p and zeros σ_z are determined by variables A, B, and C. Since A and C have been previously specified, B is the only remaining variable. If the location of the poles is of primary interest, Equation (38) will determine B exactly. Thus, as a result of realizing σ_p exactly, σ_z will be specified. This leaves no choice to the location of the zeros.

This method is restricted to the conditions of Equations (38), (39), and (40), which restricts the realizable Z_{12} function to complex-poles and zeros.

Synthesis Method IV.--Another realizable RC impedance is

$$z_b = \frac{A}{s + B} \,. \tag{41}$$

This method is actually a simplification of Synthesis Method III where C is equal to zero.

$$Z_{12} = \frac{s^2 + (2B - A)s + (B^2 + A^2 - AB)}{s^2 + (2B + A)s + (B^2 + A^2 + AB)} \,. \tag{42}$$

The location of the poles and zeros for Z_{12} are

$$s_p = -(B + A/2) \pm j\,0.866\,A \,, \tag{43}$$

$$s_z = -(B - A/2) \pm j\,0.866\,A \,. \tag{44}$$

This method can realize right-halfplane zeros if $A/2 > B$, but it is restricted to realizing identical imaginary parts for both the poles and the zeros.

Synthesis Method V.--Another realizable RC impedance is

$$z_b = \frac{s + B}{A\,s} \tag{45}$$

and from Equation (24)

$$Z_{12} = \frac{2A^2\, s(s + 1 + B)}{s^2 + 4BS + 2B^2} \,. \tag{46}$$

The location of the poles and zeros for Z_{12} are

$$s_p = -3.414\,B, \quad -0.586\,B \tag{47}$$

and

$$s_z = -(1 + B)\ . \tag{48}$$

The poles and zeros of this Z_{12} function will always be real. There is only one variable B to specify the complete Z_{12} function.

Example of Synthesis Method III.--Let

$$Z_{12} = (k)\,\frac{s^2 + 2s + 2}{s^2 + 6s + 18}\ . \tag{49}$$

The location of the poles and zeros of Z_{12} are

$$s_p = -3 \pm j\,3\ , \tag{50}$$

$$s_z = -1 \pm j\,1\ . \tag{51}$$

From Equation (40b), ω_z must be less than 3/2. Since the necessary condition for realization has been met, from Equation (35)

$$C = \frac{1 + 3 - 3}{3 - 1} = \frac{1}{2}\ , \tag{52}$$

and from Equation (37)

$$A = \frac{3}{0.433(2)} \times \frac{1}{2} = 1.732\ . \tag{53}$$

The choice of B depends on whether the pole locations σ_p or zero locations σ_z are considered the more important. After arbitrarily selecting the pole locations for this example from Equation (38)

$$B = \frac{3(1/4 + 1/2 + 1) - 1.732(1)}{1/4 + 1/2 + 1} = 2.01 \ . \tag{54}$$

Upon substituting B equal to 2.01 into Equation (39) and solving for σ_z, $\sigma_z = 2.01$. Therefore, this method has not exactly realized the Z_{12} function as desired. The coefficient associated with this Z_{12} is determined by

$$k = \frac{C^2 - C + 1}{C^2 + C + 1} \tag{55}$$

and is equal to $3/7$ in this example. The actual realized Z_{12} function is

$$Z_{12} = \frac{3}{7}\,\frac{s^2 + 4.04s + 5.04}{s^2 + 6s + 18} \tag{56}$$

and the network is shown in Figure 13.

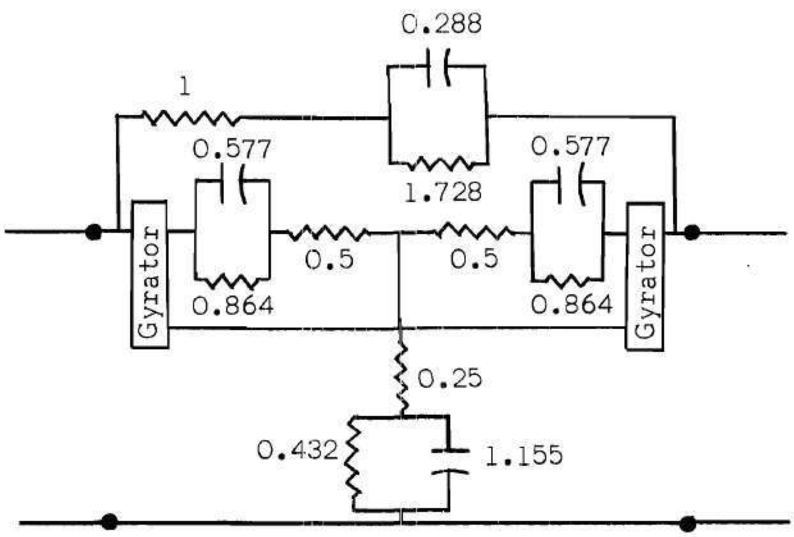


Figure 13. The Realized Z_{12} Function

CHAPTER IV

COMPARISONS AND CONCLUSIONS

Synthesis Method I is a general synthesis method which is capable of realizing a pair of unrestricted zeros and a pair of left-halfplane poles for the transfer function. In general, two negative resistors and two gyrators are required. Any condition which requires more than two negative resistors in the unbalancing branches (ref. Figure 2a) can always be removed by a wye to delta transformation. This transformation will always reduce the number of required negative resistors by at least one.

Synthesis Method II is a completely general synthesis method. It is capable of realizing a pair of unrestricted poles and zeros for the transfer function. In general this synthesis method requires two gyrators and two negative resistors. In this synthesis method the choice of the gain factor k generally determines the number of negative resistors required. As a general "rule of thumb," the best value of k is the largest value that will satisfy the conditions in Equation (11) or (12). For example, if in the first example of Synthesis Method II a gain factor k equal to 1.1 were used instead of 3, then it would require two additional negative resistors in the unbalanced equivalent circuit.

When Synthesis Method II is used to realize a transfer function that has its a_o term larger than its b_o term, generally the gain factor k can be chosen to be greater than one. Under the same conditions Synthesis Method I normally has a gain factor k that is restricted to values less than one. Therefore, since both methods generally require the same number of active elements, Synthesis Method II has a gain advantage over Synthesis Method I when a_o is greater than b_o. Conversely, when b_o is greater than a_o, usually there is not any significant advantage in gain using either method. However, the choice of the gain factor in Synthesis Method II can be done by inspection, while in Method I a trial and error procedure is sometimes required.

The condition of Equation (6) which requires that the poles of the transfer function in Synthesis Method I be in the left halfplane is not a necessary condition for realization in Synthesis Method II. The occurrence of right-halfplane poles in the transfer function may require additional negative resistors.

In Special Cases (I) and (J) the unbalanced equivalent circuits have been specified for the transfer function when it contains real and alternating poles and zeros. These two special cases have the advantage that only one gyrator is needed. When realizing real left-halfplane poles and zeros the network may or may not require negative resistance. If negative resistance is required, then the maximum number of negative resistors needed is two. When real right-halfplane poles and zeros are being realized, additional negative resistances are usually required.

Synthesis Methods III, IV, and V offer alternate procedures in unbalancing the symmetrical lattice. These synthesis methods are not general. Therefore, it is not always possible to realize a given transfer function exactly. Often an approximation has to be made.

In summing up, Synthesis Method II is more general than the method presented by Thomas using negative impedance converters. However, lattice decomposition by the use of gyrators is generally more difficult and more restrictive than lattice decomposition by negative impedance converters with the exception of Synthesis Method II.

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